

\mathcal{L} -Fuzzy Databases in Arrow Categories

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September 30, 2015

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- \mathcal{L} -Fuzziness.
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Structured Query Language and Relational Databases.

School Table	
ID	Name
S001	University of Technology
S002	University of Applied Science

Student Table			
School ID	ID	Name	DOB
S001	UT-1000	Tommy	05/06/1995
S001	UT-1000	Better	16/04/1995
S002	UAS-1000	Linda	02/09/1995
S002	UAS-1000	Jonathan	22/06/1995

Fig. 1: An example of a database table.

- A database consists of relations (tables) with attributes (columns) and unique instances with values for each attribute (rows).
- SQL is the standard language used to interact with a relational database.
- Relational databases are based on the classical set theory.

FSQL and Fuzzy Relational Databases

Crisp relational databases and SQL can not handle imprecise or vague information.

A fuzzy set maps elements to the fixed unit interval $[0,1]$.

Fuzzy databases and FSQL are based on the properties and operations of fuzzy theory.

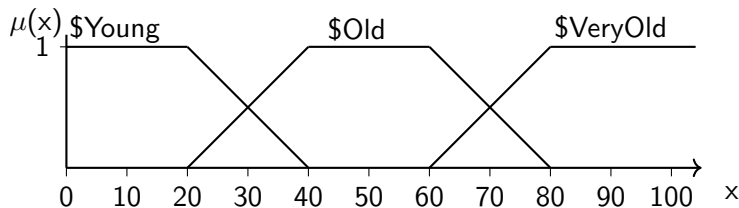


Figure: 3 Age distribution in terms of linguistic labels

Select Statement

```
SELECT Name, Age, Ability, CDEG(*)  
FROM Student  
WHERE Age NFEQ $Young THOLD 0.6  
      AND Ability FGEQ $Skilled THOLD 0.6
```

- **Linguistic Labels:** Names of fuzzy sets already stored in the Meta-knowledge base. E.g. **\$Young**, **\$Long**.
- **Fulfillment Threshold:** It represents an α - cut i.e., the condition is true if its degree is greater than the threshold.
- **Logical Connectors:** They combine two or more conditions. E.g. **AND**, **OR**.

The Elements of FSQL.

Fuzzy Comparators: They are used to compare two attributes of the same type or an attribute and a constant.

- **Possibility Comparators:** They select tuples which generally comply to a given condition.
 - In other words, the condition is true for all or some of the possible instances.
E.g. Possibly Fuzzy Greater Than (**FGT** or **F>**).
- **Necessity Comparators:** They select tuples which strictly comply to a given condition.
 - In other words, the condition is true for all possible instances.
Eg. Necessarily Fuzzy Less Or Equal To (**NFLEQ** or **NF≤**).

An Example of a Fuzzy SQL Statement.

Std_Id	Name	Age (yrs)	Height (ft)	Courses
104	Tom Benson	32	5.5	4
108	Peter Yankey	38	6.8	5
102	Eric Boadu	26	5.9	3
106	John Smith	41	6.9	4

Table 2: Students' Records.

SELECT Std_Id, Name, Course, **CDEG(*)** **FROM** Students **WHERE**
Age **FEQ \$Young 0.8** **AND** Height **FGEQ \$Tall 0.6**

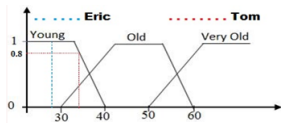


Fig. 4: Distribution of Age.

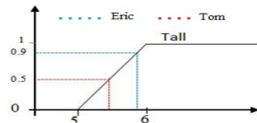


Fig. 5: Distribution of Height.

CDEG	St_ID	Name	Course
0.9	102	Eric Boadu	3

Table 3: Resulting table.

The Concept of \mathcal{L} -Fuzziness.

- It is a generalization of the fuzziness.
- An \mathcal{L} -fuzzy set maps elements to arbitrary lattice \mathcal{L} of membership degrees.
- \mathcal{L} can be $\{0, 1\}$ modeling crisp sets or $[0, 1]$ modeling the ordinary fuzzy set developed by Zadeh.
- It allows a more flexible selection of membership degrees than the unit interval $[0, 1]$.
- It gives variety of degrees of membership.
- It also comes with essential properties and operations.

The Matrix Representation of \mathcal{L} -Fuzzy Relation

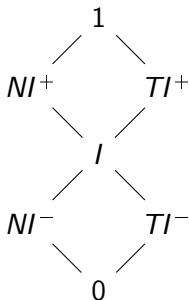


Fig. 8: Lattice structure of students' responses.

	<i>P.Sci</i>	<i>Comp</i>	<i>Geog</i>	<i>Maths</i>
<i>S1</i>	1	<i>TI</i> ⁺	<i>NI</i> ⁺	<i>TI</i> ⁻
<i>S4</i>	0	1	<i>I</i>	<i>TI</i> ⁺
<i>S3</i>	<i>NI</i> ⁺	<i>TI</i> ⁻	1	0
<i>S2</i>	0	<i>NI</i> ⁺	<i>NT</i> ⁻	1
<i>S5</i>	<i>TI</i> ⁺	1	0	<i>I</i>

Fig. 9: Representation of the students' records in \mathcal{L} -fuzzy relation.

Every entry into the \mathcal{L} -fuzzy database is an \mathcal{L} -fuzzy set.

- We can explicitly list an \mathcal{L} -fuzzy set in the database.
E.g. $\{\alpha/5.5\text{ft}, \dots, \alpha/5.9\text{ft}\}$, $\{1/25\text{yrs}, \gamma/29\text{yrs}\}$.
- We can obtain \mathcal{L} -fuzzy set through a pre-implemented characteristic function. E.g.

$$\mathbf{\$Young}(x) = \begin{cases} 1 & \text{iff } x \leq 25\text{yrs}, \\ \gamma & \text{iff } 25\text{yrs} < x \leq 29\text{yrs}, \\ \alpha & \text{iff } 29\text{yrs} < x \leq 39\text{yrs}, \\ 0 & \text{otherwise} \end{cases}$$

\mathcal{L} -Fuzzy Databases and \mathcal{L} -Fuzzy Data.

- We can obtain new fuzzy set through computation of upper bounds or lower bounds if the domain is ordered. E.g.

$$lbd(\$Old) = \$Young.$$

$$ubd(\$Old) = \$Very_Old.$$

- We can also achieve new set through approximate equality.

extremely(\equiv, s) and *very*(\equiv, s) are strengthening modifiers.

more_or_less(\equiv, s) and *roughly*(\equiv, s) are loosening modifiers.

\mathcal{L} -Fuzzy Comparators: They compare an attribute and an \mathcal{L} -Fuzzy value or attributes of the same kind.

- **Possibility Comparators** are based on general criteria.

$$\mathbf{\$Heavy} = \{..,0/64,\beta/65,.., \beta/70, \delta/71, ..,\delta/80, 1/81,.. \}.$$

Tina's Weight could be 69kg, 70kg or 71.

Is Tina's Weight **F= \$Heavy** with degree δ ? Yes.

- **Necessity Comparator** are based on all the specified criteria.

Is Tina's weight **NF= \$Heavy** with degree δ ? No.

\mathcal{L} -Fuzzy Threshold puts additional restriction on the condition in the **WHERE** clause.

- It has the word **THOLD** and comes with a degree.
- The \mathcal{L} -fuzzy threshold is optional.

Weight **F** \leq **\$Heavy THOLD** α

\mathcal{L} -Fuzzy Logical Connectors combine two or more conditions.

- The **OR** Logical Connector is basically the join operator of the given lattice \mathcal{L} .

Weight **F** \leq **\$Heavy THOLD** α **OR** Height **NF** \geq **\$Tall**

The Elements of \mathcal{L} FSQL

- The **AND** Logical Connector is basically the meet operator of the given lattice \mathcal{L} .

Weight $\mathbf{F} \leq \mathbf{\$Heavy THOLD} \alpha \mathbf{AND}$ Height $\mathbf{NF} \geq \mathbf{\$Tall}$

- We can obtain additional logical connectors through lattice-ordered semigroups.
- Such connectors could be **AND(*)** or **OR(*)**.
- The user can specify the operation he/she wants to use.

Weight $\mathbf{F} \leq \mathbf{\$Heavy THOLD} \alpha \mathbf{AND(*)}$ Height $\mathbf{NF} \geq \mathbf{\$Tall}$.

The \mathcal{L} FSQL Create Statement.

- The **CREATE** statement creates an empty table.
- It contains the table's name, attributes' names and the domains of the attributes.
- The table's name and the attributes' names must be unique in the database and the table respectively.

```
CREATE TABLE Players (Player_Id String, Name String, Height  
Float, Weight Float, Contract Int);
```

The \mathcal{L} FSQL Insert Statement

- The **INSERT** statement adds new record to an existing table.
- The values to be inserted should be \mathcal{L} -fuzzy sets.
- Each value must belong to the domain of their corresponding attribute.

INSERT INTO Players (Player_Id, Name, Height, Weight, Contract) **VALUES** (P10, Kelly, **\$Tall**, 75, 5);

- The values P10 and Kelly are crisp value i.e., \mathcal{L} -fuzzy set with one element.

The \mathcal{L} FSQL Select Statement

- The **SELECT** statement retrieves records which satisfy a given condition.
- It has the **SELECT** clause indicating which attribute(s) to retrieve the record(s) from.
- The **FROM** clause contains the table(s) from which we can compare and retrieve values from.
- The **WHERE** clause contains the condition(s) to be satisfied.

```
SELECT Player_Id, Name, Height,  
Weight FROM Players WHERE Height F= $Tall THOLD  $\gamma$  AND  
Weight NF  $\leq$  $Heavy THOLD  $\delta$ ;
```

The \mathcal{L} FSQL Delete Statement

- The **DELETE** statement acts like the **SELECT** statement.
- It drops the selected records from the table.
- It states the name of table from which the records are to be deleted and the condition(s) to be satisfied.
- The **WHERE** clause in the **DELETE** statement is the same as the one in the **SELECT** statement.

DELETE FROM Player **WHERE** Weight $F \geq$ \$Heavy **THOLD** δ ;

Example of an \mathcal{L} FSQL Statement

$$\text{\$Tall} = \{0/5.0, \dots, 0/5.4, \alpha/5.5, \dots, \alpha/5.9, \gamma/6.0, \dots, \gamma/6.4, 1/6.5, \dots\}$$

$$\text{\$Heavy} = \{0/45, \dots, 0/64, \beta/65, \dots, \beta/70, \delta/71, \dots, \delta/80, 1/81, \dots\}$$

Player_Id	Name	Height (ft.)	Weight
P10	Kelly	\\$Tall	75
P08	Thomas	5.2	\\$Heavy
P02	John	{5.4,5.5,6}	79

Table 4: Players' records.

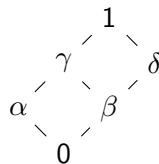


Figure: Lattice

SELECT Player_Id, Name, Height, Weight **FROM** Players **WHERE**
 Height **F** \geq **\\$Tall** γ **AND** Weight **NF** \leq **\\$Heavy** δ ;

Player Id	Name	Height (ft.)	Weight
P10	Kelly	\\$Tall	75
P02	John	{5.4,5.5,6}	79

Table 5: Resulting table.

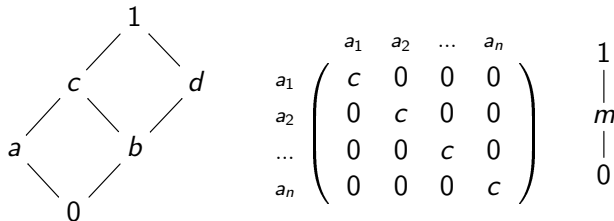
A Suitable Categorical Framework of \mathcal{L} -Fuzzy Relations

- \mathcal{L} -**Rel** defines the basic operations on \mathcal{L} -fuzzy relations
- Freyd and Scedrov introduced and then extended **allegories** as a categorical relational calculus
- Olivier and Sarrato introduced **Dedekind categories** which are equivalent to locally complete distributive allegories
- It can be shown that the class of \mathcal{L} -fuzzy relations form a Dedekind category, but it is too weak to express 0-1 crispness
- **Arrow categories** extend Dedekind categories by the addition of two arrow operations: the up-arrow (\uparrow) and the down-arrow (\downarrow)

$$\begin{pmatrix} 0.5 & 0 & 0.9 \\ 0.4 & 1 & 0 \\ 1 & 0.2 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The Arrow Category \mathcal{A} and the Interpretations

- We require that the **relational products**, **relational sums**, splittings, unit, zero object, injections, projections in \mathcal{A} to be crisp.
- In \mathcal{A} the complete **Heyting algebra** of **scalar elements** is isomorphic to \mathcal{L} . For every $I \in \mathcal{L}$ we have a scalar $I(I)$ in \mathcal{A} .



- A domain D is interpreted by an object $I(D)$ in \mathcal{A} and an element $d \in D$, by a crisp point $I(d) : \mathbf{1} \rightarrow I(D)$

$$I(\text{Richy}) = * \begin{array}{ccccc}
 \text{John} & \text{Kevin} & \text{Linda} & \text{Richy} & \text{Tijo} \\
 \left(\begin{array}{ccccc}
 0 & 0 & 0 & 1 & 0
 \end{array} \right)
 \end{array}$$

- For a natural number n , $I(n)$ denotes the object $\underbrace{1 + \dots + 1}_{n\text{-times}}$

Semantics of \mathcal{L} -Fuzzy Sets

- σ_s is the metadata database that stores \mathcal{L} sets, i.e., $\sigma_s(\$m) : 1 \rightarrow I(D)$

- For explicitly given sets: $\llbracket \{l_1/d_1, \dots, l_n/d_n\} \rrbracket (\sigma_s) = \bigsqcup_{i=1}^n I(l_i); I(d_i)$

$$\begin{aligned}
 \llbracket \{c/4, 1/5, d/6\} \rrbracket (\sigma_s) &= * \begin{matrix} & * & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ & (c) ; * & (& 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0) \sqcup \\ & & & & & & & & & & & \end{matrix} \\
 & * \begin{matrix} & * & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ & (1) ; * & (& 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0) \sqcup \\ & & & & & & & & & & & \end{matrix} \\
 & * \begin{matrix} & * & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ & (d) ; * & (& 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0) \\ & & & & & & & & & & & \end{matrix} \\
 & = * \begin{matrix} & & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ & (& 0 & 0 & 0 & c & 1 & d & 0 & 0 & 0) \\ & & & & & & & & & & & \end{matrix}
 \end{aligned}$$

- For linguistic labels: $\llbracket \$m \rrbracket (\sigma_s) = \sigma_s(\$m)$

$$\llbracket \$Close \rrbracket (\sigma_s) = \sigma_s(\$Close) = * \begin{matrix} & & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ & (& 1 & 1 & 1 & 1 & 1 & c & a & 0 &) \\ & & & & & & & & & & & \end{matrix}$$

Semantics of Tables

In the non-fuzzy case a table R can be seen as a finite subset of $D_1 \times \cdots \times D_n$. Relation algebraically this can be modelled by

- a point $\llbracket R \rrbracket : 1 \rightarrow \mathcal{P}(I(D_1) \times \cdots \times I(D_n))$, where $\mathcal{P}(X)$ is an abstract version of a power set construction
- a vector $\llbracket R \rrbracket : 1 \rightarrow I(D_1) \times \cdots \times I(D_n)$
- a function $\llbracket R \rrbracket : I(r) \rightarrow I(D_1) \times \cdots \times I(D_n)$ since we deal with finite sets.

In fuzzy database, the target object within last option becomes $\mathcal{P}(I(D_1)) \times \cdots \times \mathcal{P}(I(D_n))$ which isomorphic to $\mathcal{P}(I(D_1) + \cdots + I(D_n))$

Having a function of the form $\llbracket R \rrbracket : I(r) \rightarrow \mathcal{P}(I(D_1) + \cdots + I(D_n))$ is equivalent to having a relation of the form $\llbracket R \rrbracket : \mathbf{I}(r) \rightarrow \mathbf{I}(D_1) + \cdots + \mathbf{I}(D_n)$.

Semantics of Tables (cont.)

A	B
a	d
b	c

(A) A classical database

A	B
{m/a, 1/b}	{0/c, m/d}
{0/a, m/b}	{1/c, 0/d}

(B) An \mathcal{L} -fuzzy database

$$\mathcal{L}^A \times \mathcal{L}^B$$

$$\left(\begin{array}{cc} \{m/a, & \{0/c, \\ 1/b\} & m/d\} \end{array} \right)$$

(C) $A \times B$ \mathcal{L} -subset

$$\begin{array}{c} s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6 \quad \dots \quad s_{80} \quad s_{81} \\ 1 \left(\begin{array}{cccccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \\ 2 \left(\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

(D) The table as a crisp relation (to $\mathcal{L}^A \times \mathcal{L}^B$)

$$\mathcal{L}^{A+B}$$

$$\left\{ \begin{array}{c} m/a \\ 0/c \\ 1/b \\ m/d \end{array} \right\}$$

(E) $A + B$ \mathcal{L} -subset

$$\begin{array}{c} T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \quad \dots \quad T_{80} \quad T_{81} \\ 1 \left(\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \\ 2 \left(\begin{array}{cccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

(F) The table as a crisp relation (to \mathcal{L}^{A+B})

$$\begin{array}{c} a \quad b \quad c \quad d \\ 1 \left(\begin{array}{cccc} m & 1 & 0 & m \\ 0 & m & 1 & 0 \end{array} \right) \\ 2 \left(\begin{array}{cccc} m & 1 & 0 & m \\ 0 & m & 1 & 0 \end{array} \right) \end{array}$$

(G) The table as an \mathcal{L} -relation

Figure: Semantics of \mathcal{L} -fuzzy tables

Semantics of Tables (cont.)

- σ_t maps a table name to its semantics, i.e., a relation of the form $\llbracket R \rrbracket : I(r) \rightarrow I(D_1) + \dots + I(D_n)$
- Also, $\sigma_t[Q/R]$ denotes the update of the database at table R by the relation Q . Mathematically,

$$\sigma_t[Q/R](X) = \begin{cases} Q, & \text{if } X = R, \\ \sigma_t(X), & \text{otherwise.} \end{cases}$$

- Selecting an attribute: $\llbracket R.A_i \rrbracket(\sigma_s, \sigma_t) = \llbracket R.A_i \rrbracket(\sigma_t) = \sigma_t(R); \iota_i^\sim$

$$= \begin{matrix} & a & b & c & d \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} m & 1 & 0 & m \\ 0 & m & 1 & 0 \end{pmatrix} \end{matrix}; \begin{matrix} & a & b & c & d \\ \begin{matrix} a \\ b \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix} \smile$$

$$= \begin{matrix} & a & b & c & d \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} m & 1 & 0 & m \\ 0 & m & 1 & 0 \end{pmatrix} \end{matrix}; \begin{matrix} & a & b \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \end{matrix} = \begin{matrix} & a & b \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} m & 1 \\ 0 & m \end{pmatrix} \end{matrix}.$$

Semantics of INSERT Statement

Let us say $\{m_1, \dots, m_n\}$ are \mathcal{L} -fuzzy subsets of the domains (D_1, \dots, D_n) . We define the semantics for a tuple of that table by

$$\llbracket (m_1, \dots, m_n) \rrbracket (\sigma_S) = \bigsqcup_{i=1}^n \llbracket m_i \rrbracket (\sigma_S); \iota_i.$$

Example:

$$\begin{aligned} \llbracket (\{1/a, m/b\}, \{0/c, 1/d\}) \rrbracket (\sigma_S) &= \llbracket \{1/a, m/b\} \rrbracket (\sigma_S); \iota_1 \sqcup \llbracket \{0/c, 1/d\} \rrbracket (\sigma_S); \iota_2 \\ &= * \begin{pmatrix} a & b \\ 1 & m \end{pmatrix}; \begin{matrix} a & b & c & d \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix} \sqcup * \begin{pmatrix} c & d \\ 0 & 1 \end{pmatrix}; \begin{matrix} a & b & c & d \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \\ &= * \begin{pmatrix} a & b & c & d \\ 1 & m & 0 & 0 \end{pmatrix} \sqcup * \begin{pmatrix} a & b & c & d \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ &= * \begin{pmatrix} a & b & c & d \\ 1 & m & 0 & 1 \end{pmatrix} \end{aligned}$$

Semantics of INSERT Statement (cont.)

In order to deduce the semantics for the final table where this tuple has been added, we refer to the following figure.

$$\begin{array}{ccc}
 r & & \\
 \downarrow \iota & \searrow \sigma_t(R) & \\
 r + 1 & \xrightarrow{\llbracket R(m_1, \dots, m_n) \rrbracket(\sigma_s, \sigma_t)} & I(D_1) + \dots + I(D_n) \\
 \uparrow \kappa & \nearrow \llbracket (m_1, \dots, m_n) \rrbracket(\sigma_s) & \\
 1 & &
 \end{array}$$

- The table has r rows: $I(r) \rightarrow I(D_1) + \dots + I(D_n)$
- A vector $1 \rightarrow I(D_1) + \dots + I(D_n)$ represents the tuple to be inserted
- The resultant table has $r + 1$ tuple and maps from the object $r + 1$ to $I(D_1) + \dots + I(D_n)$ in \mathcal{A} .

$$\llbracket R(m_1, \dots, m_n) \rrbracket(\sigma_s, \sigma_t) = \iota^\smile; \sigma_t(R) \sqcup \kappa^\smile; \llbracket (m_1, \dots, m_n) \rrbracket(\sigma_s)$$

Semantics of INSERT Statement: An Example

$$\llbracket R(m_1, \dots, m_n) \rrbracket(\sigma_s, \sigma_t) = \iota^\smile; \sigma_t(R) \sqcup \kappa^\smile; \llbracket (m_1, \dots, m_n) \rrbracket(\sigma_s)$$

$$\llbracket R(\{1/a, m/b\}, \{0/c, 1/d\}) \rrbracket(\sigma_s, \sigma_t)$$

$$= \begin{matrix} & 1 & 2 & 3 \\ 1 & \left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix} \right)^\smile; & 1 & \left(\begin{matrix} a & b & c & d \\ m & 1 & 0 & m \\ 0 & m & 1 & 0 \end{matrix} \right) \sqcup & 1 & \begin{matrix} * \\ \left(\begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \right) \end{matrix}; & \begin{matrix} a & b & c & d \\ \left(\begin{matrix} 1 & m & 0 & 1 \end{matrix} \right) \end{matrix} \end{matrix}$$

$$= \begin{matrix} & 1 & 2 \\ 1 & \left(\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} \right); & 1 & \left(\begin{matrix} a & b & c & d \\ m & 1 & 0 & m \\ 0 & m & 1 & 0 \end{matrix} \right) \sqcup & 1 & \begin{matrix} * \\ \left(\begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \right) \end{matrix}; & \begin{matrix} a & b & c & d \\ \left(\begin{matrix} 1 & m & 0 & 1 \end{matrix} \right) \end{matrix} \end{matrix}$$

$$= \begin{matrix} & a & b & c & d \\ 1 & \left(\begin{matrix} m & 1 & 0 & m \\ 0 & m & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix} \right) \sqcup & 1 & \left(\begin{matrix} a & b & c & d \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & m & 0 & 1 \end{matrix} \right) = & 1 & \left(\begin{matrix} a & b & c & d \\ m & 1 & 0 & m \\ 0 & m & 0 & 1 \\ 1 & m & 0 & 1 \end{matrix} \right) \end{matrix}$$

Semantics of INSERT Statement (cont.)

Note that the first injection ι maps from the r -ary relational sum of the unit object to the $(r + 1)$ -ary relational sum. In our example, as the table already contains 2 tuples and so the final table would have 3 tuples, ι injects to the object $I(3)$.

Finally, we define the semantics of an INSERT statement as follows:

$$\begin{aligned} & \llbracket \text{INSERT INTO } R \text{ VALUES } (m_1, \dots, m_n); \rrbracket (\sigma_s, \sigma_t) \\ &= \sigma_t [\llbracket R(m_1, \dots, m_n) \rrbracket (\sigma_s, \sigma_t) / R]. \end{aligned}$$

From the definition of update function it is clear that the new relation replaces the existing one for the particular table.

Thank You