CSI — introductory exercises to AI-based reasoning

Exercise 1. Check the relational type

$$\mathsf{succ} \xleftarrow{\mathsf{length}} (a{:})$$

where $\mathsf{length} : A^* \to \mathbb{N}_0$ is the well-known function that computes the length of a finite list and $\mathsf{succ}\ n = n + 1$.

Exercise 2. Show that function $f^\star$, defined by $f^\star\ x = [f\ a \mid a \leftarrow x]$, is length-**invariant**.

Exercise 3. Find a **simulation** for $f^\star$ taking $\mathsf{length} : A^* \to \mathbb{N}_0$ as **abstraction** function.

Exercise 4. Consider the predicate $nempty$ that tests if a finite list is empty or not: $nempty = (>0) \cdot \mathsf{length}$.

Suppose you wanto to show that $(a{:})$ preserves $nempty$:

$$nempty \xleftarrow{(a:)} nempty \tag{1}$$

Use abstract interpretation (over length) to show that the obvious fact

$$\langle \forall\ n\ :\ n > 0 :\ \mathsf{succ}\ n > 0 \rangle$$

is enough for (1) to hold.

Exercise 5. Suppose operation S simulates R via h and Q simulates P (via k), that is,

$$S \xleftarrow{h} R$$
$$Q \xleftarrow{k} P$$

hold. Show that compound operation $S + Q$ simulates $R + P$ via compound abstraction function $h + k$.